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Using Wearout Information to Reduce Reliability Demonstration Test Time

by

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Upper and lower bounds are obtained for the amount of linear test time required to demonstrate a specified lower confidence limit value on reliability when time to failure has a Weibull distribution with known shape parameter. These bounds are compared with the corresponding required test time when failure time has an exponential distribution. Equations are developed for the ratios of these times.

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Using Wearout Information to Reduce Reliability Demonstration Test Time

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Abstract

Upper and lower bounds are obtained for the amount of linear test time required to demonstrate a specified lower confidence limit value on reliability when time to failure has a Weibull distribution with known shape parameter. These bounds are compared with the corresponding required test time when failure time has an exponential distribution. Equations are developed for the ratios of these times.

Key Words: Reliability testing; Demonstration testing; Life testing; Weibull life testing

1. PROBLEM STATEMENT AND ANALYSIS

The time (hours) to failure, T, of a device is assumed to have a Weibull distribution with reliability function $R(t)$ given by

$$P(T > t) = R(t) = e^{-(\lambda t)^{\beta}} \tag{1}$$

It is well known that T^{β} has an exponential distribution with failure rate λ^{β}. That is,

$$P\left(T^{\beta} > t\right) = e^{-\lambda^{\beta} t}. \tag{2}$$

If $\beta = 1$, T has an exponential distribution.

A $100\gamma\%$ lower confidence limit (LCL) of at least R_0 is specified for the reliability at given time, t_0 hours. The following reliability demonstration test is performed to validate this requirement assuming $\beta = 1$.

> Items are tested sequentially until failure or the sum of their test times equals a given value, T_0 hours. T_0 is chosen so that if exactly f failures occur during $(0, T_0)$, the derived LCL value for $R(t_0)$ will equal R_0. The demonstration test is administered by testing items until a total of T_0 hours has been accumulated on the items tested in sequence at which time testing stops and the number of failures, F, observed. If $F \le f$, the LCL specification of R_0 is validated.

It is well known that the $100\gamma\%$ LCL, $R(t_0)_L$, for $R(t_0)$ under time censored life testing as described above is

$$R(t_0)_L = e^{-\lambda_u t_0} \tag{3}$$

where

$$\lambda_u = \frac{\chi^2_{\gamma,2(1+f)}}{2T_0} \tag{4}$$

and $\chi^2_{\gamma,n}$ is the 100γ percentile point of the chi-square distribution with n degrees of freedom. Solving for T_0 in the equation

$$e^{-\lambda_u t_0} = R_0 \tag{5}$$

yields

$$T_0 = \frac{t_0\, \chi^2_{\gamma,2(1+f)}}{-2\ln R_0} \equiv \text{ELT}. \tag{6}$$

Equation (4) follows directly from the following two facts:

1. The number of failures F in (0, T_0) under the above described test plan has a Poisson distribution with mean $\alpha = \lambda T_0$;

2. The $100\gamma\%$ upper confidence limit (UCL), α_u, for the mean, α, of a Poisson distribution using one observation, F, is (Reference 1)

$$\alpha_u = \frac{\chi^2_{\gamma,2(1+F)}}{2}.. \tag{7}$$

Suppose $\beta \neq 1$ is known and the same $100\gamma\%$ LCL value of R_0 has been specified for $R(t_0)$. The reliability demonstration test is modified as follows:

> Items are tested sequentially until failure and the sum of their test times raised to the β power equals a given value, T_0^β (T_0 in hours). T_0 is chosen so that if exactly f failures occur during $\left(0, T_0^\beta\right)$, the derived LCL value for $R(t_0)$ will equal R_0. If fewer than f failures occur during the test time accumulation period $\left(0, T_0^\beta\right)$, the LCL specification of R_0 is validated.

Since $R(t_0) = e^{-(\lambda t_0)^\beta}$, the $100\gamma\%$ LCL, $R(t_0)_L$, for $R(t_0)$ is

$$R(t_0)_L = e^{-\left(\lambda^\beta\right)_u t_0^\beta} \tag{8}$$

where $\left(\lambda^\beta\right)_u$ is the $100\gamma\%$ UCL for λ^β.

Since T_i^β has an exponential distribution for each failed item, the number of failures, F, in $\left(0, T_0^\beta\right)$ has a Poisson distribution with mean $\lambda^\beta T_0^\beta$. Consequently the $100\gamma\%$ UCL, $\left(\lambda^\beta\right)_u$, has the form

$$\left(\lambda^\beta\right)_u = \frac{\chi^2_{\gamma,2(1+F)}}{2T_0^\beta}. \tag{9}$$

Setting $F = f$ in equation (9), solving for T_0 in the equation

$$e^{-\left(\lambda^{\beta}\right)_u t_0^{\beta}} = R_0 \tag{10}$$

yields the equation

$$T_0 = t_0 \left(\frac{\chi^2_{\gamma,2(1+f)}}{-2 \ln R_0} \right)^{\frac{1}{\beta}}. \tag{11}$$

It is important to note that the actual linear test time that will be accumulated under this test plan is random. The linear test time, WLT, if f failures occur, is

$$\text{WLT} = T_1 + T_2 + \ldots + T_f + T_{f+1} \tag{12}$$

where

$$\sum_{L=1}^{f+1} T_i^{\beta} = T_0^{\beta}. \tag{13}$$

T_i = failure time of item i, $i = 1, 2, \ldots, f$

and T_{f+1} = residual time after failure number f.

If $\beta > 1$, WLT in equation (12) has the minimum value of T_0 if $T_1 = T_0$ or if $f - 1$ items fail at time 0 and $T_f = T_0$. That is,

$$\text{Min WLT} \equiv \text{mWLT} = T_0. \tag{14}$$

Since

$$T_{f+1} = \left(T_0^{\beta} - \sum_{i=1}^{f} T_i^{\beta} \right)^{\frac{1}{\beta}} \tag{15}$$

equation (12) can be written as

$$\text{WLT} = g\left(T_1, T_2, \ldots, T_f\right) = \sum_{1}^{f} T_i + \left(T_0^{\beta} - \sum_{i=1}^{f} T_i^{\beta} \right)^{\frac{1}{\beta}}. \tag{16}$$

If $\beta > 1$, the maximum value of WLT can be obtained by taking the f partial derivatives of WLT in equation (16) and equating them to zero. Doing so yields the following set of equations.

$$1 - T_i^{\beta-1}\left(T_0^{\beta} - \sum_{i=1}^{f} T_i^{\beta}\right)^{\frac{1}{\beta}-1} = 0, \quad i = 1,2,\ldots,f. \tag{17}$$

Consequently,

$$T_i = \left(T_0^{\beta} - \sum_{i=1}^{f} T_i^{\beta}\right)^{\frac{1}{\beta}} = T_{f+1}, \quad i = 1,2,\ldots,f. \tag{18}$$

That is, $T_1 = T_2 = \ldots = T_{f+1}$.

Since

$$\begin{aligned} T_0^{\beta} &= \sum_{i=1}^{f+1} T_i^{\beta} = (f+1)T_i^{\beta}, \quad i = 1,2,\ldots,f \\ T_i &= T_0 \big/ (f+1)^{1/\beta}. \end{aligned} \tag{19}$$

That is, the maximum value of WLT occurs when all T_i equal the value in equation (19); i.e.,

$$\begin{aligned} \mathrm{MWLT} = \max \mathrm{WLT} = \max \sum_{i=1}^{f+1} T_i &= (f+1)\frac{T_0}{(f+1)^{1/\beta}} \\ &= T_0 (f+1)^{1-\frac{1}{\beta}}. \end{aligned} \tag{20}$$

Substituting T_0 from equation (11) into equation (20)

$$\mathrm{MWLT} = t_0 \left(\frac{\chi^2_{\gamma,2(1+f)}}{-2\ln R_0}\right)^{\frac{1}{\beta}} (f+1)^{1-\frac{1}{\beta}}. \tag{21}$$

The ratio of equation (21) to equation (6) yields

$$\frac{\mathrm{MWLT}}{\mathrm{ELT}} = \left(-\frac{(f+1)\ln R_0}{K(\gamma,f)}\right)^{1-\frac{1}{\beta}} \tag{22}$$

where

$$K(\gamma, f) = \frac{\chi^2_{\gamma, 2(1+f)}}{2}.$$

Likewise,

$$\frac{\text{mWLT}}{\text{ELT}} = \frac{T_0}{\text{ELT}} = \left(-\frac{\ln R_0}{K(\gamma, f)}\right)^{1-\frac{1}{\beta}}. \qquad (23)$$

Tables 1 and 2 display some values of these two ratios for selected values of R_0, β, and γ. If $\beta < 1$, mWLT is the maximum value of WLT and MWLT in equation (21) is the minimum value of WLT. This is relected in Tables 1 and 2 for $\beta = .8$ and .9.

Table 1

VALUES OF (MWLT/ELT, mWLT/ELT) FOR γ = .80

	$R_0 = .90$				$R_0 = .95$			
β	$f=1$	$f=2$	$f=3$	$f=4$	$f=1$	$f=2$	$f=3$	$f=4$
.8	1.94, 2.31	1.92, 2.52	1.90, 2.69	1.89, 2.83	2.32, 2.76	2.30, 3.02	2.28, 3.22	2.26, 3.38
.9	1.34, 1.45	1.34, 1.51	1.33, 1.55	1.33, 1.59	1.45, 1.57	1.45, 1.63	1.44, 1.68	1.44, 1.72
1.1	0.79, 0.74	0.79, 0.71	0.79, 0.70	0.79, 0.69	0.74, 0.69	0.74, 0.67	0.74, 0.65	0.74, 0.64
1.2	0.64, 0.57	0.65, 0.54	0.65, 0.52	0.65, 0.50	0.57, 0.51	0.57, 0.48	0.58, 0.46	0.58, 0.44
1.3	0.54, 0.46	0.55, 0.43	0.55, 0.40	0.56, 0.38	0.46, 0.39	0.46, 0.36	0.47, 0.34	0.47, 0.32
1.5	0.41, 0.33	0.42, 0.29	0.42, 0.27	0.43, 0.25	0.32, 0.26	0.33, 0.23	0.33, 0.21	0.34, 0.20
2.0	0.27, 0.19	0.27, 0.16	0.28, 0.14	0.28, 0.13	0.19, 0.13	0.19, 0.11	0.19, 0.10	0.20, 0.09

Table 2

VALUES OF (MWLT/ELT, mWLT/ELT) FOR γ = .90

	$R_0 = .90$				$R_0 = .95$			
β	$f=1$	$f=2$	$f=3$	$f=4$	$f=1$	$f=2$	$f=3$	$f=4$
.8	2.07, 2.46	2.03, 2.67	2.00, 2.82	1.97, 2.95	2.48, 2.95	2.43, 3.19	2.39, 3.38	2.36, 3.53
.9	1.38, 1.49	1.37, 1.55	1.36, 1.59	1.35, 1.62	1.50, 1.62	1.48, 1.67	1.47, 1.72	1.47, 1.75
1.1	0.77, 0.72	0.77, 0.70	0.78, 0.69	0.78, 0.67	0.72, 0.67	0.72, 0.66	0.73, 0.64	0.73, 0.63
1.2	0.62, 0.55	0.62, 0.52	0.63, 0.50	0.64, 0.49	0.55, 0.49	0.55, 0.46	0.56, 0.44	0.56, 0.43
1.3	0.51, 0.43	0.52, 0.40	0.53, 0.38	0.53, 0.37	0.43, 0.37	0.44, 0.34	0.45, 0.33	0.45, 0.31
1.5	0.38, 0.30	0.39, 0.27	0.40, 0.25	0.40, 0.24	0.30, 0.24	0.31, 0.21	0.31, 0.20	0.32, 0.19
2.0	0.23, 0.16	0.24, 0.14	0.25, 0.13	0.26, 0.11	0.16, 0.11	0.17, 0.10	0.18, 0.09	0.18, 0.08

2. SUMMARY COMMENTS

The values in Tables 1 and 2 are rather surprising. For example, at most 57% of the linear test time required to demonstrate an 80% LCL value of 0.95 for reliability under an exponential test plan ($\beta = 1$) is required to demonstrate the same requirement if $\beta = 1.2$. This strongly suggests that it is likely to be worth the effort to look for data on items similar to the ones being tested that would provide a reasonably accurate estimate of the shape parameter under similar test conditions. Such an effort could lead to a conservative choice of β that is larger than 1 which would save significant test time compared to test time required when the exponential distribution is assumed. Of course, choosing a value for β greater than the actual value would lead to an optimistic LCL estimate for $R(t_0)$, assuming the Weibull distribution is the appropriate distribution in the first place.

The equations developed in this report clearly demonstrate that assuming there is no wearout (exponential distribution) when there is wearout (Weibull distribution, $\beta > 1$) in reliability demonstration testing using time censored test plans, can result in a potential lost opportunity to save significant resources.

The small values of the ratios in Tables 1 and 2 also suggest that if significant wearout is thought to be present, and the Weibull distribution is thought to be an appropriate distribution for failure times, then testing under the Weibull assumption with both parameters unknown may be less costly than using test plans developed under an exponential assumption. Considerably more comprehensive analysis is needed to formulate this hypothesis and determine the consequent conditions for its validity.

Reference

1. Bain, L. J. and M. Engelhardt, *Introduction to Probability and Mathematical Statistics*, 2nd Edition, Duxbury Press, 1992.

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